9-1 day 7 Power Series

Learning Objectives:

I can identify the function that a power series models

I can write a power series that models a given function

I can use integration and differentiation to determine the power series of a function given the power series of another function

I can determine the radius and interval of convergence of a power series

Jan 31-2:08 PM

Consider the series:

$y = \frac{1}{1-x}$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots$$

Geometric

$a_1 = 1$

$r = x$

converge $|r| < 1$

diverge $|r| \geq 1$

$-1 < r < 1$

$-1 < x < 1$

$S = \frac{a_1}{1-r}$

$$\boxed{S = \frac{1}{1-x}}$$

$x = 1/2$ $S = 2$

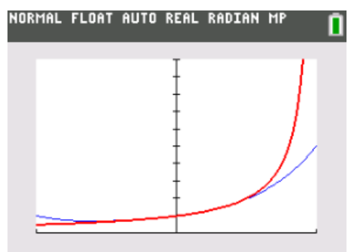
$x = 4/5$ $S = 5$

$x = .1$ $S = 10/9$

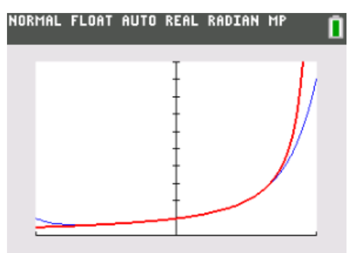
x	S
$1/2$	2
$4/5$	5
$.1$	$10/9$

```
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW
 Xmin=-1
 Xmax=1
 Xscl=1
 Ymin=0
 Ymax=10
 Yscl=1
 Xres=1
 ΔX=.00757575757575
 TraceStep=.01515151515151
```

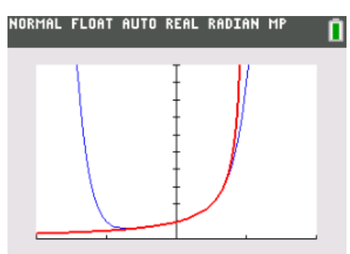
```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
\Y1=1+X+X²+X³+X⁴
\Y2=1/(1-X)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```




```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
\Y1=1+X+X²+X³+X⁴+X⁵+X⁶+X⁷+X
\Y2=■/(1-X)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=
```




```
NORMAL FLOAT AUTO REAL RADIAN MP
DISTANCE BETWEEN TICK MARKS ON AXIS
WINDOW
 Xmin=-2
 Xmax=2
 Xscl=■
 Ymin=0
 Ymax=10
 Yscl=1
 Xres=1
 ΔX=.01515151515151
 TraceStep=.03030303030303
```



Jan 31-2:08 PM

Power Series

An expression of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_n x^n + \ldots$$

is a Power Series centered at x = 0.

An expression of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + \ldots + c_n (x-a)^n + \ldots$$

is a Power Series centered at x = a.

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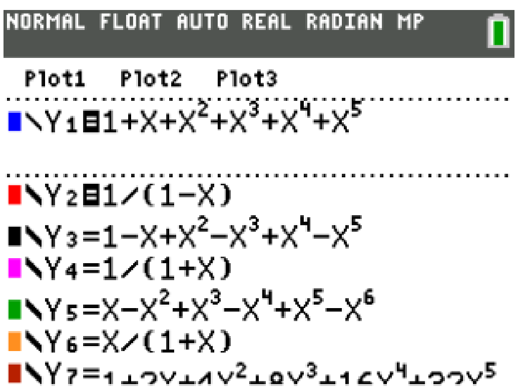
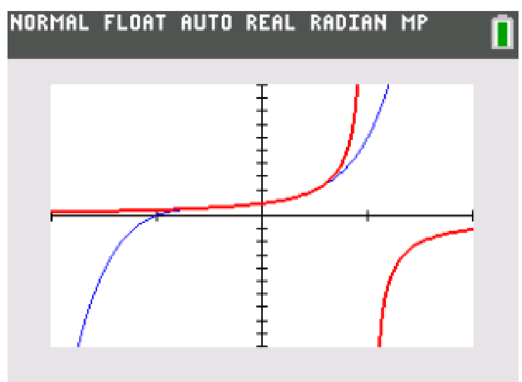
In Groups, do exploration #1 on page 477

$$1 + x + x^2 + x^3 + x^4 + \cdots$$

$$\sum_{n=0}^{\infty} x^n \Longrightarrow \frac{1}{1-x}$$





Jan 31-2:08 PM

1. $\frac{1}{1+x} = \frac{1}{1-(-x)}$

$1 + -x + x^2 + -x^3 + x^4 + \cdots$

$\sum_{n=1}^{\infty} (-x)^n$

2. $\frac{x}{1-(-x)} = x - x^2 + x^3 - x^4 + \cdots$

$\sum_{n=1}^{\infty} -(-x)^n$

3. $\frac{1}{1-2x}$ $\quad 1 - 2x + 4x^2 - 8x^3 + \cdots$

$\sum_{n=0}^{\infty} (2x)^n$

4. $\frac{1}{x} \Rightarrow \frac{1}{1-(-x+1)}$ $\quad a_1 = 1$, $r = -x+1$

$1 + (-x+1) + (-x+1)^2 + (-x+1)^3 + \cdots$

$\sum_{n=0}^{\infty} (-x+1)^n$

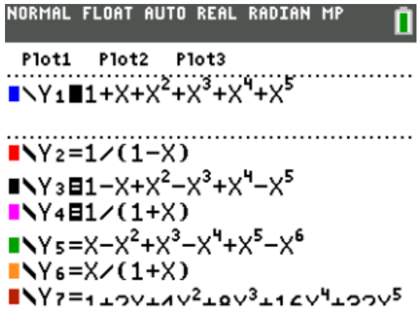
5. $\frac{1/3}{1-(-x+1)}$ $\quad a_1 = 1/3$, $r = (-x+1)$

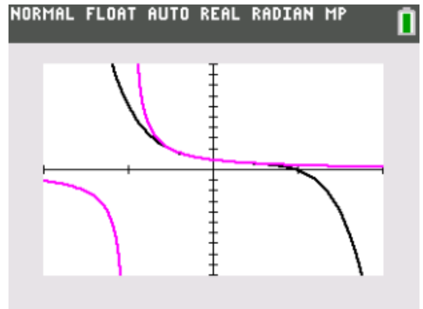
$\frac{1}{3} + \frac{1}{3}(-x+1) + \frac{1}{3}(-x+1)^2 + \frac{1}{3}(-x+1)^3 + \cdots$

$\sum_{n=0}^{\infty} \frac{1}{3}(-x+1)^n$

Feb 27-12:33 PM

1.





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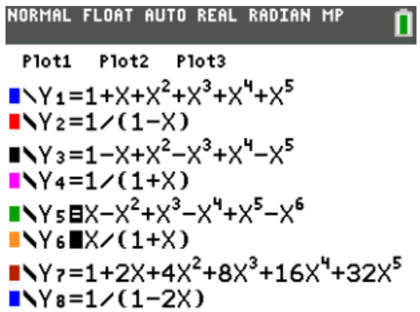
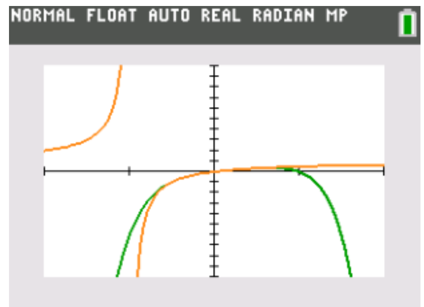
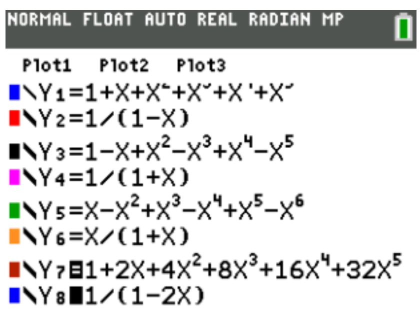
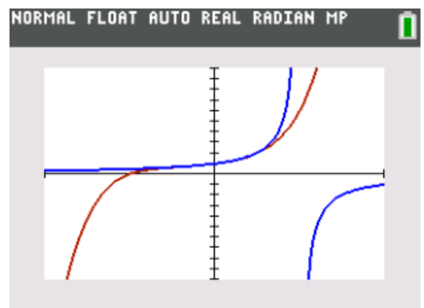


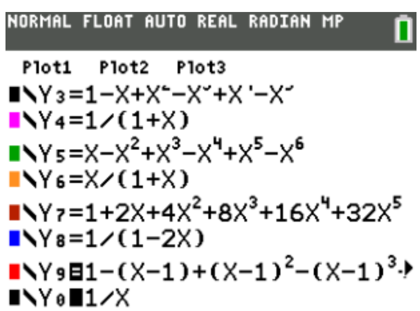


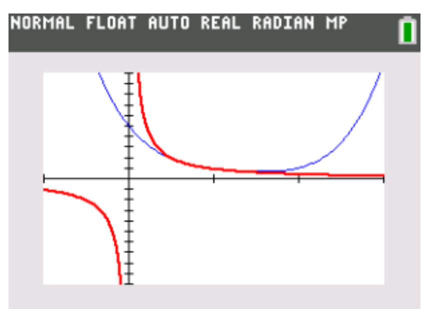
3.

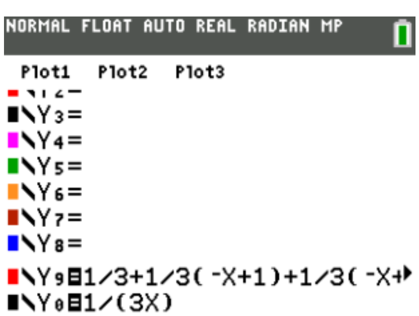


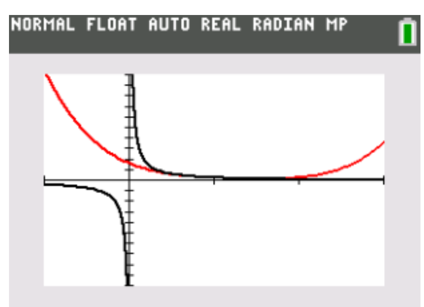


4.









Feb 27-11:39 AM

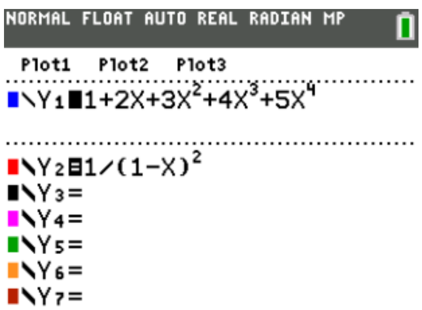
Ex1. Given: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$

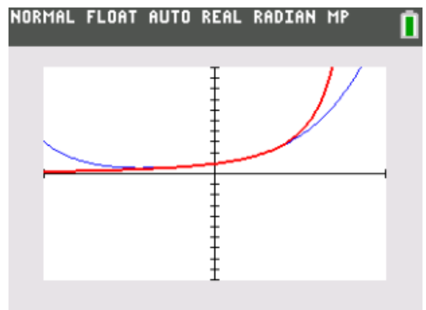
a.) Differentiate both sides to find the power series for another function.

$(1-x)^{-1}$

$-(1-x)^{-2} \cdot -1$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$

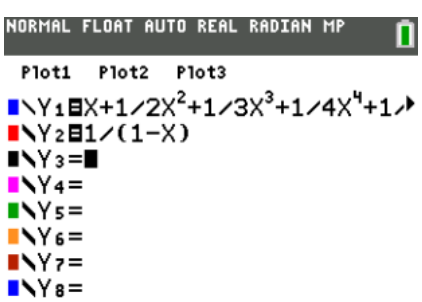


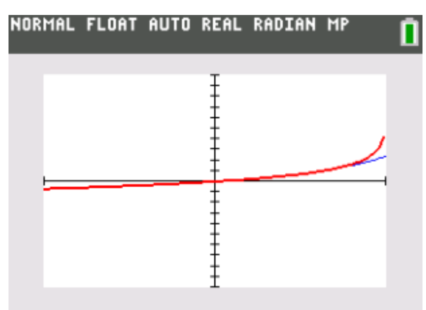


b.) Integrate both sides to find the power series for another function.

$\int \frac{1}{1-x}\,dx = \int 1 + x + x^2 + x^3 + \ldots + x^n + \ldots \, dx$

$-\ln|1-x| = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \ldots$





Jan 31-2:08 PM

In Groups, do exploration #2 on page 480

$$\boxed{y=\frac{1}{1+x^2}}$$

$$y=\frac{1}{1-(-x^2)} \qquad a_1=1 \quad r=-x^2$$

$$1-x^2+x^4-x^6+x^8+\cdots$$

$$\sum_{n=0}^{\infty}(-x^2)^n$$

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

Y1 = $1-X^2+X^4-X^6+X^8-X^{10}+X^{12}$

Y2 = $1/(1+X^2)$

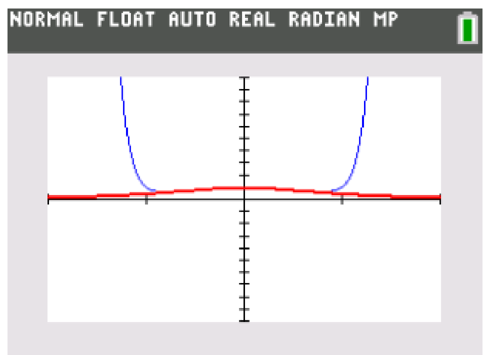
Y3 =

Y4 =

Y5 =

Y6 =

Y7 =



$$y=\tan^{-1}x$$

$$x-\frac{1}{3}x^3+\frac{1}{5}x^5+-\frac{1}{7}x^7+\frac{1}{9}x^9+\cdots$$

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

Y1 = $X-1/3X^3+1/5X^5-1/7X^7$

Y2 = $\tan^{-1}(X)$

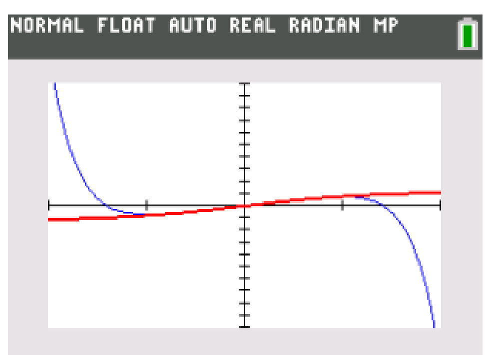
Y3 =

Y4 =

Y5 =

Y6 =

Y7 =



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In Groups, do exploration #3 on page 480 (do #1-3, 6, 7)

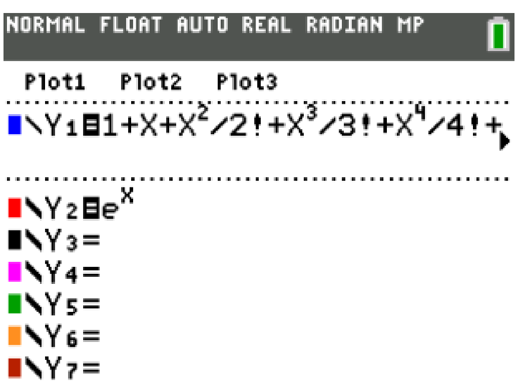
$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

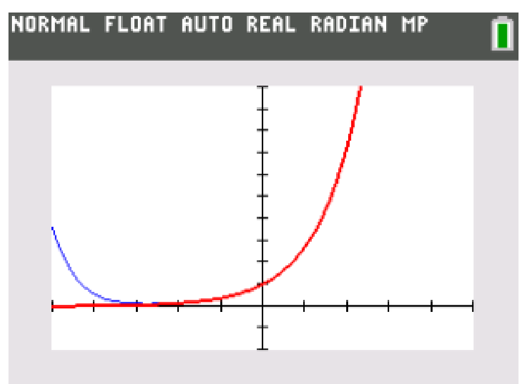
$$\boxed{f(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots}$$

$$f'(x) = 1 + \frac{2}{2!}x + \frac{3}{3!}x^2 + \frac{4}{4!}x^3 + \dots$$

$$f'(x) = 1 + \frac{\cancel{2}}{\cancel{2}\cdot 1}x + \frac{\cancel{3}}{\cancel{3}\cdot 2!}x^2 + \frac{\cancel{4}}{\cancel{4}\cdot 3!}x^3 + \dots$$

$$\boxed{f'(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots}$$





Homework

Pg 481 # 21-34,
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